GRASERS BASED ON PARTICLE ACCELERATORS AND ON LASERS

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Abstract

Grasers based on a stimulated emission of gravitational radiation by relativistic charged particle beams in external fields and on conversion of laser radiation into gravitational one in the magnetic fields as well as detectors are discussed. A scheme of the gravitational radiation not accompanied by an useless inaccessible by a value average power of the electromagnetic radiation and stimulation of the conversion of gravitons into photons in gravitational detectors by an open resonator are considered.

1 Introduction

The contribution of the gravitational interaction to other kinds of interactions between elementary particles is very small. That is why to investigate the nature of the gravitational field and it's interaction with another fields (conversion of gravitons into photons and other particles and so on) we are forced to use pure gravitational radiation for this purpose. On the way to this goal first of all we must prove the real existence of gravitational waves and study theirs properties. Then we could solve the problem of the choice of a proper relativistic theory of gravity, to continue the development of this theory and to verify it's predictions through new experiments [1]. If it will be possible one day the next step could be the investigation of the significance of the gravity in the elementary particle physics.

Nowadays the efforts on experimental investigation of foundations of the theory of gravity are concentrated on the detection of gravitational radiation coming from the universe. However the wavelengths of the existing natural sources of the gravitational radiation like double stars is very high (period is about 1 day). The intensity of these and other more hard natural gravitational radiation sources is very small [2]. Phenomena similar to Supernovae explosions (when much higher power and frequency ($\sim 1kHz$) gravitational radiation is emitted) don't often happen. That is why we are forced to produce an artificial source of hard gravitational radiation. One of the possible versions of such source is the gravitational analogue of Laser named a Graser. The necessity in Grasers is similar to that in nuclear research when particle accelerators began to be used instead of cosmic rays.

In this paper both Grasers based on a direct emission of gravitational waves by nonuniformly moving prebunched ion beams in undulators (parametric Free-Ion Grasers) and Grasers based on the conversion of gaussian laser beams stored in an open resonator into gravitational beams in a transverse magnetostatic fields are considered. It was shown that one of optimal solutions of the problem of a Graser based on direct emission of gravitational radiation by particles can be a Graser based on an undulator, a cutoff waveguide arranged in the undulator and a relativistic heavy ion beam. Such Graser is working under conditions when a gravitational radiation is not accompanied by the useless electromagnetic radiation with an inaccessible by a value average power. In the relativistic case these conditions are valid at wavelengths much less then the cutoff one. An effective method of stimulation of the conversion process of gravitons into photons in a transverse magnetic field, multilayer mirrors and other systems of gravitational detectors by a resonator are considered.

2 Emission of gravitational radiation

Weak gravitational processes in a weak gravitational field of the Earth can be described in the Euclidian space. In static the Coulomb's law of forces between electric charges and Newton's law of the universal gravitation have the same dependence of forces on a distance R between particles ($\sim 1/R^2$) and on theirs charges e and masses M. It means that in the case of gravitational field we can introduce the gravitational charge $e_{gr} = \sqrt{G}\,M$ and use it in the static gravity theory the same way as the electric charge is used in the electrostatic, where $G = 6.67\,10^{-8}$ is the gravitational constant. The difference in the formulas of these lows is only in the sign. Gravitational field is always attractive field for all particles.

We can extend this analogy from static to dynamic of charges, forces and fields and

suppose that gravitational field of moving particle in general case (that is in an arbitrary reference frame and arbitrary particle motion) will have both the analogy of the electric $\vec{E}^{gr}$ and analogy of the magnetic $\vec{B}^{gr}$ field strengths determined by a gravitational charge e_{gr}, velocity and acceleration. If we proceed from this supposition and from the equivalence of the inertial reference frames then in order to this analogy was fruitful we must determine the dynamical lows of gravity. At that we can proceed from the analogy with the dynamical approach to the construction of the special theory of relativity (G.Lorentz, A.Poincare) [3]. The simplest low can be found by considering some particular problems with known solution. In particular we know that a system of 2 identical particles with masses M and in a charge state n^+ will be in equilibrium in any arbitrary moving coordinate system when it is in equilibrium in a coordinate system at rest ($G\,M^2 = (n^+)^2e^2$). If the gravitational force is higher ($G\,M^2 > (n^+)^2e^2$) then such system will be in the state of rotation. At that in accordance with the relativistic transformations the longitudinal dimension of the orbits of particles will be compressed and the period of rotation will be increased relative to the laboratory coordinate system when the center of mass is moving with a relativistic velocity. It means that the electromagnetic and gravitational transformations of forces and fields must be the same and the equations of gravity must be similar to the Maxwell's equations [4], [5]. Otherwise the laws of the particle motion will have different forms in different rest frames moving with different velocities if gravitational, electromagnetic and any other forces between particles will have different transformation properties.

So in a general case time-varying week gravitational analogies of electric and magnetic fields in the first approximation are described by the wave equations which are similar to the wave equations of the electromagnetic fields (with the accuracy to signs of terms of the equations which include the density of charges). The solution of the gravitational equations for the nonuniformly moving particle which undergo the influence of the extraneous fields will lead to the gravitational analogies of the electromagnetic fields of the form similar to Lienard-Viechert's form for the electromagnetic fields.

It follows that the nonuniformly moving charged particle will emit the gravitational radiation with the same parameters (except power) as the electromagnetic radiation of these particles in free space (the influence of the boundary conditions for the gravitational radiation is negligible). That is why we can declare that Free-Particle Lasers will emit stimulated gravitational radiation simultaneously with the stimulated electromagnetic radiation. These kinds of radiation emitted in the conditions of free space will have identical spectral and angular distributions[1]. The ratio of the power of the gravitational to electromagnetic radiation

[1]According to this concept we can draw the conclusion that all sources of the electromagnetic radiation (thermal sources, lasers, klystrons and so on) are the sources of the gravitational radiation simultaneously. The difference between emission of electrons in undulator of Free-Electron Laser and in an atom is in the fact that a nucleon emit gravitational wave of the same amplitude as an electron but opposite polarity (theirs accelerations have opposite directions and inverse to theirs masses and the emitted field strengths are proportional to theirs masses and accelerations). As the wavelength of the emitted radiation in this case is much higher then the dimension of the atom then the difference between phases of the emitted radiation and π is negligible and the power of the emitted radiation will be suppressed. It will appear in the approximation $\beta^2 = (v/c)^2$, where v, c are the velocities of the electron and light accordingly. In this case in the correlation (1) a coefficient $C_\alpha = C_{\alpha 0} \simeq \beta^2 \ll 1$ will appear. However in the case of an extended media photons emitted by an atom can be absorbed by another atoms of media with high probability and gravitons will go away from media with very small absorption. That is why for the case of lasers based on high quality resonators and without extraction of the emitted electromagnetic radiation or large sources like stars the ratio of the

will be the square of the ratio of their's charges: $P^{gr}/P^{em} = e_{gr}^2/(n^+e)^2 = GM^2/(n^+e)^2$. Different gravitational theories lead to ratio

$$\frac{P^{gr}}{P^{em}} = \frac{C_\gamma GM^2}{(n^+)^2 e^2} \tag{1}$$

with the coefficient $C_\gamma \simeq 1 \div 3$. For example in the case of Einstein's theory of gravity $C_\gamma \simeq 13/4$ [6]. The values $GM^2/e^2 = 2.402 \cdot 10^{-43}$ for electron mass $m_e = 9.11 \cdot 10^{-28}$ g and $GM^2/e^2 = 7.98 \cdot 10^{-37}$ for unit atomic mass $M_u = 1.66 \cdot 10^{-24}$ g.

The flow of the monochromatic gravitons of the frequency ω_{gr}

$$\dot{N}^{gr} = \frac{P^{gr}}{\hbar\omega_{gr}} = \frac{C_\gamma GM^2}{(n^+)^2 e^2 \hbar\omega_{gr}} P^{em} = \frac{C_\gamma GM^2}{(n^+)^2 e^2} \dot{N}^{em}, \tag{2}$$

where $\hbar$ is the Plank reduced constant, $\dot{N}^{em} \simeq 5 \cdot 10^{22} P^{em} \lambda$ [ph/W·cm] the flow of the laser photons, $\lambda = 2\pi c/\omega_{gr}$.

2.1 *Grasers based on particle emission in undulators*

The total energy, hardness and directivity of the gravitational radiation of particles emitted in the magnetic fields of undulators is strongly increased with the increase of the relativistic factor γ. The number of the emitted both photons and gravitons in this case does not depend on γ. Free-Ion Lasers based on proton and more heavy ion beams of storage rings can emit more powerful coherent electromagnetic and gravitational radiation then Free-Electron Lasers of the same relativistic factor [7] - [10]. This is because of the total energy of ion beams stored in ion storage rings is much higher then the total energy of electron beams stored in electron rings. Grasers based on ion beams according to (1) have an additional advantage as the ratio of the gravitational to electromagnetic powers of the emitted radiation is proportional to the square of mass. This is important as a weak power of the gravitational radiation will be accompanied by the extremely high useless power of the electromagnetic radiation. The value of the power of the electromagnetic radiation have the inaccessible high value for electrons and light ions. Very hard conditions are in the case of heavy ions up to $^{238}_{92}U^{1+}$ and more heavy formations as well. For example if we accept the power of the electromagnetic radiation emitted by an ion beam $P^{em} = 10^8$ W, $M = 238M_u$, $n^+ = 1$, $C_\gamma = 13/4$, $(C_\gamma GM^2_{^{238}_{92}U^{1+}}/e^2 = 1.47 \cdot 10^{-31})$ then the power of the gravitational radiation is $P^{gr} = 1.47 \cdot 10^{-23}$ W, the flow of gravitons $\dot{N}^{gr} = 7.35 \cdot 10^{-5}$ gr/s when the energy of the gravitons is $\hbar\omega_{gr} = 1.25eV = 2 \cdot 10^{-19}$ J ($\lambda_{gr} \sim 1\mu$) and $\dot{N}^{gr} = 0.74$ gr/s when the energy of the gravitons is $\hbar\omega_{gr} = 1.25 \cdot 10^{-4}eV = 2 \cdot 10^{-23}$ J ($\lambda_{gr} \sim 1cm$). The production of the same flow of gravitons in the $\lambda \leq 1$ cm, region by Grasers based on Free-Electron Lasers ($C_\gamma G m_e^2/e^2 = 7.81 \cdot 10^{-43}$) require an expense of average power $1.88 \cdot 10^{11}$ times higher. This is not practicable solution.

Production of the flow of the gravitational radiation ~ 1 gr/s is not enough to detect them now. To produce higher flows we need the way of the gravitational beam production which is not accompanied by the emission of the electromagnetic radiation of the inaccessible by the value average power. One of possible solutions of this problem is in an using of

gravitational to electromagnetic radiation will be much higher ($C_\alpha \gg C_{\alpha 0}$).

the cutoff waveguides (regime of cutoff or evanescent modes [11]). In this case only the gravitational radiation will be emitted (it does not interact with waveguide).

A scheme of a Graser and at the same time a scheme of a Parametric (Prebunched) Free-Ion Laser based on an undulator and prebunched Ion Beam of a storage ring is presented on the Fig.1. Parametric Free-Ion Lasers have an advantage on ordinary Free-Ion Lasers originating from the prebunched nature of ion beams[2].

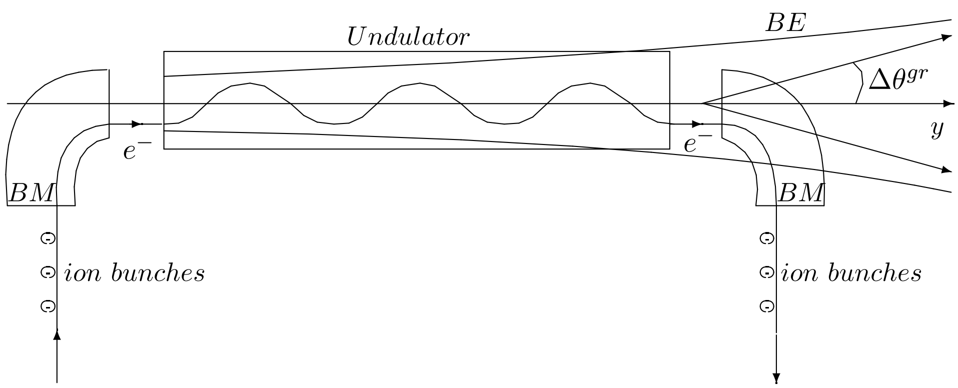

Fig.1: A scheme of the parametric Free-Ion Laser; BM: Banding Magnets; BE: Beam Envelope of the electromagnetic and gravitational radiation; e^- electron bunches.

Below we will consider the possible parameters of such Grasers. First of all we will present some consequences from the theory of Lasers and Grasers.

The minimum wavelength λ_n of the electromagnetic radiation emitted by a relativistic charged particle in an undulator on the n-th harmonic in free space and in a waveguide is defined by an undulator period λ_u, relativistic factor γ, deflecting parameter $p_\perp$ and the cutoff wavelength of the waveguide mode λ_c:

$$\lambda_n^{fs} = \frac{\lambda_u}{2n\gamma^2}(1 + p_\perp^2),$$

$$\lambda_n^{wg} = \frac{\lambda_u \left(1 \pm \sqrt{1 - (1 + p_\perp^2)(1 + \lambda_u^2/n^2\lambda_c^2)/\gamma^2}\right)}{\overline{\beta_y} n\,(1 + \lambda_u^2/n^2\lambda_c^2)}, \qquad (3)$$

where $\overline{\beta_y} = \overline{(\beta^2 - \beta_\perp^2)^{1/2}}$, $p_\perp = \left(\overline{|\vec{p}_\perp|^2}\right)^{1/2} = \left(\overline{|\vec{B}_\perp|^2}\right)^{1/2}/B_c$, $\vec{p}_\perp = \gamma\vec{\beta}_\perp$, $\vec{\beta}_\perp = \vec{v}_\perp/c$, $\vec{v}_\perp$ is a transverse velocity of the particle, $\vec{B}_\perp$ a transverse magnetic field strength of an undulator, $B_c = 2\pi Mc^2/n^+ e\lambda_u$, e an electron charge, n^+ a number of an ion charge state, a coefficient $2\pi Mc^2/e \simeq 19.56(M/M_u)[MG \cdot cm]$ [14].

[2]In ordinary Free-Ion Lasers rather long distance is necessary before a homogeneous ion beam will be bunched when starting from noise (self amplified stimulated emission) or week external electromagnetic wave. Only after bunching the ion beam can effectively produce radiation. Moreover the degree of the beam bunching in these cases is not high. Because of this factors the Parametric Free-Particle Lasers based on high degree bunched beams have limiting parameters for the rate of the energy loss. They will emit monochromatic, polarized, diffractionally limited radiation as well [7] - [10], [12], [13].

According to (3) the electromagnetic radiation will be emitted in the waveguide when the relativistic factor of particles $\gamma > \sqrt{(1+p_\perp^2)(1+\lambda_u^2/\lambda_c^2)}$. In this case the wavelengths of the emitted radiation $\lambda < \lambda_{max}^{wg}$ where

$$\lambda_{max}^{wg}|_{n=1} = \frac{\lambda_u \lambda_c^2}{\lambda_u^2 + \lambda_c^2}|_{\lambda_u \gg \lambda_c} \simeq \frac{\lambda_c^2}{\lambda_u} \simeq \frac{\lambda_c \sqrt{1+p_\perp^2}}{\gamma}. \tag{4}$$

The emitted radiation is diffraction limited. The angular divergence of the gravitational beam $\Delta\theta^{gr}$ is determined by the relativistic factor γ, the emitted wavelength λ, the particle beam radius $\sigma_\perp$ and the number of the undulator periods K:

$$\Delta\theta^{gr} = min(\frac{\lambda}{\sigma_\perp}, \frac{1}{\gamma}\sqrt{\frac{1+p_\perp^2}{nK}}). \tag{5}$$

The power emitted by an ion beam on the first harmonic in a Parametric Free-Ion Laser based on a helical undulator $(\left(\overline{|\vec{B}_\perp|^2}\right)^{1/2} = B_\perp)$ in the regime of free space when using a particle beam consisting of a series of short $(< \lambda_1)$ microbunches with a small transverse dimensions $\sigma_p \ll \lambda\gamma/\sqrt{1+p_\perp^2}$, spaced an integral number of wavelengths λ_1, apart is,

$$P^{em\,fs} = \frac{\pi^2}{c}\frac{p_\perp^2}{1+p_\perp^2}Ki^2 \qquad \text{or} \qquad P^{em\,fs} = 296.1\frac{p_\perp^2}{1+p_\perp^2}Ki^2[\frac{W}{A^2}], \tag{6}$$

where i is the electron beam current [8], [9]. When the wide beam of the radius $\sigma_p \gg \lambda\gamma/\sqrt{1+p_\perp^2}$ is used then the power tend to (6) at the distances $l_p = 2\pi\sigma_p^2/\lambda_1$ from the beginning of the undulator.

Accordingly, the power of the gravitational radiation and the flow of gravitons in the case of $C_\gamma = 13/4$ are:

$$P^{gr} = 7.68 \cdot 10^{-34}\frac{p_\perp^2}{1+p_\perp^2}(\frac{M}{M_u})^2\frac{Ki^2}{(n^+)^2}, \quad \dot{N}^{gr} = 4.79 \cdot 10^{-15}\frac{p_\perp^2}{1+p_\perp^2}(\frac{M}{M_u})^2\frac{Ki^2}{(n^+)^2\hbar\omega}, \tag{7}$$

where P is in Watts, i in Amperes and $\hbar\omega$ in eV [8], [9].

Notice that in the relativistic case the power of the emitted coherent radiation (7) does not depend on the energy of particles (γ) and the flow of gravitons $\dot{N}^{gr} \sim \gamma^{-2}$.

The value $\lambda_{max}|_{\lambda_u \gg \lambda_c} \ll \lambda_c$. It means that we can generate hard gravitational radiation $\lambda \ll \lambda_c$ under conditions when it does not accompany the electromagnetic one. The emission of higher harmonics in this Graser must be absent as well. Undulator with an ideal helical magnetic field will satisfy this condition when particle bunches emit radiation in phase in the direction of the axis.

Example 1. The beam of ${}^{238}_{92}U^{1+}$ ions pass through the waveguide which has the form of a smooth pipe of the radius $r = 1.5$ cm. The waveguide is installed into a helical undulator with a period $\lambda_u = 46.55$ m, a number of periods 16 (a length 745 m) and magnetic field strength $B_\perp = 300$ k G. The energy of ions in the storage ring is $\varepsilon_i \simeq 11.1$ TeV ($\gamma = 50.4$) and the current of the ion beam is $i = 1$ kA.

In this case the fundamental H_{11} mode has a cutoff wavelength $\lambda_c \simeq 3.41\ r \simeq 5.1$ cm, $\lambda^{wg}_{max} \simeq 5.6 \cdot 10^{-3}$ cm, $B_c \simeq 10^6$ G, $p_\perp \simeq 0.3$, $n = 1$. According to (3), (5), (6) the wavelength of the coherent radiation is $\lambda_1 \simeq 1$ cm, the energy of a graviton $1.25 \cdot 10^{-4}$ eV, the angular divergence of the gravitational beam is $\Delta\theta \simeq 5 \cdot 10^{-3}$, the power of the gravitational radiation and the flow of the gravitons emitted by the ion beam are: $P^{gr} = 5.7 \cdot 10^{-23}$ W, $\dot{N}^{gr} = 2.92$ gr/s.

If the energy of ions in the storage ring is $\varepsilon_i \simeq 57.5$ TeV ($\gamma = 260$) then the wavelength of the coherent radiation is $\lambda_1 \simeq 3.76 \cdot 10^{-2}$ cm, the energy of a graviton $3.32 \cdot 10^{-3}$ eV, the power of the gravitational radiation and the flow of the gravitons emitted by the ion beam are: $P^{gr} = 2.4 \cdot 10^{-23}$ W, $\dot{N}^{gr} = 0.11\,\text{gr/s}$, $\Delta\theta \simeq \cdot 10^{-3}$.

In these two cases $\lambda > \lambda^{wg}_{max}$ the unwanted power of the electromagnetic radiation ($P^{em} = 46.9\,\text{MW}$) will not be emitted.

Spectra of both electromagnetic and gravitational radiation sources are limited from the short wavelength region by the bunch length. In modern storage rings electron bunches of 0.3 mm (1-ps) length were achieved. The longer-term objective is the realization of 100-fs bunches [15] - [17]. In linear accelerators using electron guns with both thermal and photoelectron cathodes and bunchers the production of electron bunches of the length $\sim 0.1 \div 0.01$ mm was achieved [18] - [20]. Bunchers based on undulators and lasers permit to produce a long train of more short microbunches. The bunching frequency in this case is very high (the distance between bunches is equal or multiple to the wavelength of the laser beam). To prepare ion beams for Grasers in ion storage rings the same beam bunching problems can be solved by the same or special bunching technique [21]. Instead of a three dimensional Synchrotron Radiation Damping using in electron storage rings a Radiative Ion Cooling based on the process of resonance Rayleigh scattering of a laser light by relativistic ions [22], [9], [10] or ordinary Laser Cooling under conditions of synchro-betatron resonance [23], [24], [25] can be used in ion storage rings. Ion cooling may make it possible to store very high current low-emittance ion beams using multiple injection of ions in high energy storage rings. The beam stored energy of the LHC will exceed the value 500 MJ (average current $\sim 0.5A$). Multiple injection at the top energy of the dedicated ion storage ring will permit to increase this value. The energy stored in the proton beam at the top energy of the ISR has reached $5 \cdot 10^6$ J, in the form of a beam of 50 A at 31.5 GeV. The peak currents in excess of 10^3 A have been focused to submillimetric spots without problems [27].

2.2 *Grasers based on conversion of a laser beam into a gravitational one in a transverse magnetostatic field*

In section 2 we introduced gravitational charge e_{gr} for a particle with a mass M. We can do the next step and suppose that all forms of fields enable a density of energy $\rho = w/c^2$ and a corresponding charge density $\rho_{gr} = \sqrt{G}\rho$. In the case of electromagnetic fields $w = (|\vec{E}|^2 + |\vec{H}|^2)/8\pi$, where $\vec{E}$, $\vec{H}$ are the electric and magnetic field strengths. If the density of the energy of the electromagnetic field is varied with time then a varying with time gravitational charge density will appear and the conditions for the emission of the gravitational radiation can appear. For example, the superposition of static electromagnetic field and electromagnetic wave in vacuum will lead to a formation of a modulated gravitational charge density propagating together with electromagnetic wave the same direction with

light velocity and to emission of gravitational wave.

Electrogravitational conversion of an electromagnetic radiation to gravitational one was known to Whittaker as early as 1947 [28]. Gertsenshtein was the first to actually calculate the conversion efficiency of a plane electromagnetic wave into gravitational one in a transverse magnetic field [29], [30]. The conversion of free electromagnetic wave into gravitational one is lacking but in the simplest case of two counter propagating waves it take place [33], [34].

The way based on the conversion of the electromagnetic laser beam into the gravitational one in a one-directional transverse magnetic field is presented on Fig.2. On this figure an open resonator presented by two mirrors M_1, M_2. The resonator is located outside of the magnet poles. The distance between mirrors L_m is higher then the length of the magnet L. It can be exited by continuous beam of Free Electron or Free-Ion Laser. Conversion of electromagnetic beam into gravitational one takes place in the magnetic field. The envelop of the gravitational beam coincides with the mode envelop of the electromagnetic radiation in the resonator. Outside of the resonator the electromagnetic radiation is absent but the gravitational radiation is propagated in the limits of the prolonged resonator mode envelop.

The ratio of the power of the gravitational radiation to the active power of the electromagnetic radiation stored in the resonator that is the probability $p_{\gamma\to g}$ of the conversion

$$p_{\gamma\to g} = \frac{P^{gr}}{P_a^{em}} = \frac{G}{c^4} Q\left(\int_0^L B_\perp(y)dy\right)^2, \tag{8}$$

where $G/c^4 \simeq 8.24 \cdot 10^{-50}[1/\mathrm{cm}^2\,\mathrm{Gauss}^2]$, Q is the quality of the resonator, L the length of the magnet, $B_\perp$ the value of the transverse magnetic field [29].

It follows from the equation (8) that the effective conversion takes place in the case of single sign transverse component of the magnetic field $(\int B_\perp dy)^2 = \overline{B_\perp^2} L^2)$. The gravitational radiation will not be emitted in the magnetic field of the undulator because of in the undulator $\int B_\perp dy = 0$ (the waves emitted in the magnetic poles of undulator of different polarity will have phase shift π).

The mode distribution at an arbitrary point of resonator is determined by the expression

$$\sigma = \sigma_0\sqrt{1 + \frac{y}{l_R}}, \tag{9}$$

where $l_R = \pi\sigma_0^2/\lambda$ is the Rayleigh length, σ_0 the radius of the waist of a photon beam. In the case of confocal resonator: $L_m = 2l_R$ and $\sigma_0 = \sqrt{\lambda L_m/2\pi}$. Photon and gravitational beams radii are increased $\sqrt{2}$ times and theirs area 2 times per the length l_R. At the distance σ the energy density of beams in the transverse direction decay e times [32].

The following example illustrates possible parameters of such Graser.

Example 2. Let the value of the magnetic field is $B_\perp = 10^5$ G, the length of the bending magnet $L = 1$ km, the active losses in the cavity $P_a^{em} = 10^5$ W, the quality of the cavity $Q = 10^3$, the reactive power (which determine the conversion process) is $P_r^{em} = 100$ MW.

In this case according to (8), (9) the power of the gravitational radiation is $P^{gr} = 8.26 \cdot 10^{-22}$ W, the flow of the gravitons and the radius of the waist of the photon (graviton) beams are $\dot{N}^{gr} = 4 \cdot 10^{-3}$ gr/s, $\sigma_0 = 1.26$ cm and $\dot{N}^{gr} = 40$ gr/s, $\sigma_0 = 126$ cm for the photon

energy 1.25 eV and $1.25 \cdot 10^{-4}$ eV accordingly. The main problem in this scheme is the heating problem of mirrors. The figures above can be increased by increasing of the length of the magnet, the value of the magnetic field if it will be possible in future developments of the superconducting technique and the quality of resonators in the centimeter and lower wavelength regions.

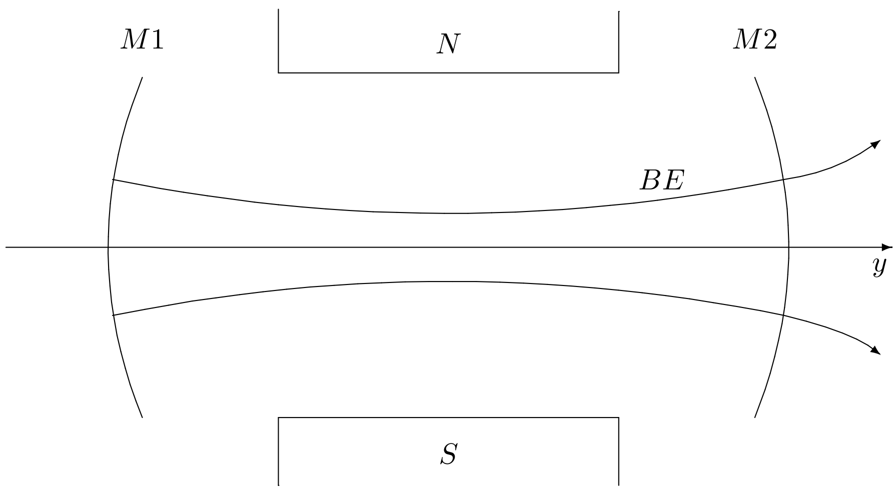

Fig.2: A scheme of conversion of an electromagnetic laser beam exited in an open resonator into gravitational one; M_1, M_2: Mirrors; N, S: magnetic poles; BE: Beam Envelops of an electromagnetic and gravitational radiations.

3 Detectors of the gravitational radiation

The schemes of detectors of artificial sources of gravitational waves can be based both on the coherent emission of the electromagnetic radiation by charged particle beams or another charged formations accelerated by the gravitational waves and on an inverse conversion of gravitons into photons in the transverse magnetic field. Multilayer mirrors can be created from charged electron and ion beams in storage rings (moving multilayer mirrors) [21] or from transparent charged dielectric layers placed along the way of the gravitational beam.

3.1 *Detectors based on coherent conversion of gravitons into photons by ion bunches (backward Gravi-Compton scattering)*

The conversion cross section of the graviton to photon on a charged ion is determined by the equation

$$\sigma = \sigma_i \frac{C_\gamma G M_i^2}{e^2} = \sigma_e \frac{C_\gamma G m_e^2 (n^+)^2}{e^2} \tag{10}$$

where $\sigma_i = \sigma_e (m_e/M_i)^2 (n^+)^2$ is the ion scattering cross section, $\sigma_e = 8\pi r_e^2/3 \simeq 6.65 \cdot 10^{-25}\, cm^2$ Thomson scattering cross section. Notice that the cross section (10) does not depend on mass of particle but depend on ion charge state n^+.

In the case of prebunched ion beam in the storage ring (moving multilayer ion mirror) the coherent cross section σ_{coh} per one ion will be the cross section (10) multiplied by the coherence factor which is equal $6M_bN_1$ for the point like bunches (transverse dimensions less then $\lambda\gamma$, longitudinal dimensions less then λ), where M_b is the number of bunches in the straight section determined by the length of the straight section L and the wavelength λ_{rf} of the radio frequency system of the storage ring ($M_b = L/\lambda_{rf}$), $N_1 = i\lambda_{rf}/ecn^+ \simeq 2.08 \cdot 10^8\lambda_{rf}i[A]/n^+$ the number of ions in one ion bunch, $\lambda = \lambda_{rf}/4\gamma^2$ is the wavelength of the backward scattered electromagnetic radiation (process: graviton + ion → photon + ion), γ the relativistic factor of ions. The coefficient 6 includes the coefficient 2 which appeared from the fact that in the relativistic case each graviton will meat $2M_b$ ion bunches for the time of crossing of the straight section and coefficient 3 which take into account the coherence between bunches. It follows that in this case the coherent cross section is equal

$$\sigma_{coh} = 6M_bN_1\sigma \simeq 6.47 \cdot 10^{-58}M_bi[A]\lambda_{rf}[cm]/n^+. \tag{11}$$

The luminosity of the storage ring under consideration

$$L_{gi} = \frac{2iL\dot{N}^{gr}}{ecS_{gr}} \simeq 4.17 \cdot 10^8 i[A]L[sm]/S_{gr}[cm^2]. \tag{12}$$

where S_{gr} is the area of the gravitational beam.

In the case of i = 1kA, λ_{rf} = 1 cm, ($N_1 = 2.08 \cdot 10^{11}$), $M_b = 10^4$ (L = 100m), $n^+ = 1$, $S_{gr} = 10^4 cm^2$: $\sigma_{coh} \simeq 6.47 \cdot 10^{-51}$ cm^2, $L_{gi} = 4.17 \cdot 10^{11}$ cm^{-2} s^{-1}, and the flow of the backward scattered photons $\dot{N}^{bs} = 2.7 \cdot 10^{-39}$ ph/s. The energy of scattered radiation will be determined by the energy of the dedicated storage ring and at ($\gamma \simeq 2 \div 5$) will lay in the submillimeter and more hard regions. This scheme permit to increase the energy of the emitted photons the more the higher the energy of the storage ring. High quality ($Q \gg 1$) open resonator can be used to increase the low of photons Q^2 times inside and Q times outside of the resonator (extracted).

3.2 *Detectors based on conversion of gravitons into photons by a charged multilayer mirror*

The coefficient of reflection of electromagnetic waves by a thin layer of electrons is equal $k_r = r_e^2N_\sigma^2\lambda^2$, where N_σ is the electric charge surface density, λ the wavelength of the falling radiation [35]. When electrons are arranged in M_l layers then in this case the maximal coefficient of reflection will be M_l^2 times higher.

By analogy the gravitational waves will lead to an acceleration of electrons of the multilayer mirror and hence to emission of electromagnetic waves. The converted electromagnetic waves will propagate the same direction as the reflected electromagnetic waves. The conversion coefficient of the gravitational radiation to electromagnetic one in this case will be less then the reflection coefficient of the electromagnetic waves in accordance with the coefficient $C_\gamma Gm_e^2/e^2$:

$$k_{ml} = C_\gamma Gm_e^2r_e^2N_\sigma^2\lambda^2M_l^2/e^2. \tag{13}$$

On practice transparent dielectric media can be cut off on thin flat layers. These layers can be charged from both sides by electric charges of the opposite sign. They can be

arranged such a way that the surfaces with identical signs of charges was brought together as shown on Fig.3.

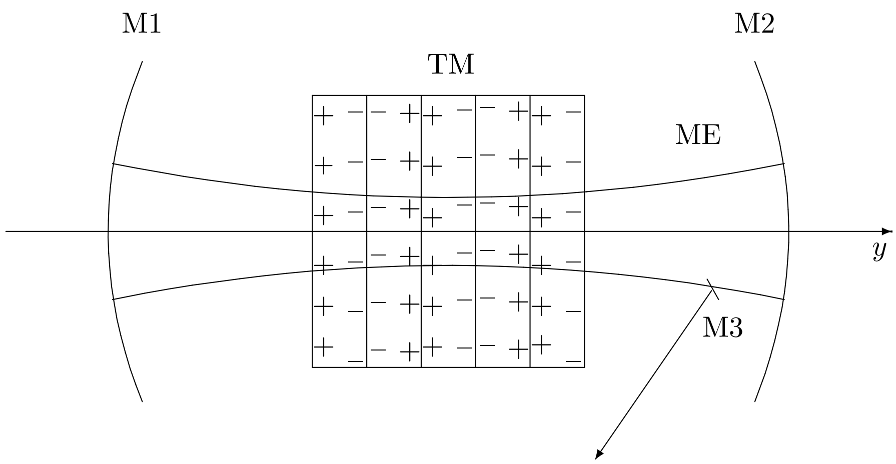

Fig.3: A scheme of conversion of a gravitational beam into electromagnetic one; M_1, M_2: Mirrors; BE: Beam Envelop of an electromagnetic radiation; M3: Mirror for extraction of radiation.

The propagating in the direction of the axis "y" gravitational radiation will be converted into electromagnetic radiation by charged layers. The open resonator with a quality Q will stimulate the conversion process Q times. As a result the flow of photons will be increased Q^2 times inside and Q times outside of the system. The phases of the waves emitted by layers with opposite polarities will differ on π. The velocity of gravitational waves are equal to light velocity in any case but the velocity of the electromagnetic waves will depend on the refraction index of the media. That is why we can choose the distance between layers such a way that the emitted electromagnetic waves will be in phase forward or backward directions.

In this scheme the surface density of electric charge will be limited by the dielectric breakdown electric field strength E_{br} ($E_{br} \leq 6 \cdot 10^3$ or $\leq 2 \cdot 10^6$ [V/cm])

$$N_\sigma = \frac{E_{br}}{2\pi e} \simeq 1.1 \cdot 10^6 E_{br}[V/cm]. \tag{14}$$

According to (13), (14) the efficiency of the system

$$\frac{\dot{N}_r^{em}}{\dot{N}^{gr}} = k_{ml} \cdot Q^2 \simeq 7.51 \cdot 10^{-56} \lambda_{rf}^2 M_l^2 E_{br}^2 Q^2. \tag{15}$$

where $\dot{N}_r^{em}$ is the flow of the converted photons stored inside the resonator. The flow of photons extracted from the resonator is Q times less.

In the case of $\lambda_{rf} = 1cm$, $M_l = 10^5$ ($L \simeq 1\,\mathrm{km}$), $Q = 10^9$, $E_{br} = 2$MV/cm, the efficiency $\dot{N}_r^{em}/\dot{N}^{gr} = 3 \cdot 10^{-15}$. The flow of photons $\dot{N}_r^{em} = 3 \cdot 10^{-12}$ ph/s when $\dot{N}^{gr} = 10^3$ gr/s. This value is much higher then in the previous one.

3.3 *Detectors based on conversion of gravitons into photons by charged mirrors of an open resonator*

If in the scheme of Fig.3 the media in a resonator is absent but mirrors $M1$ and $M2$ are charged then such system will be excited. The efficiency of such system will be much less then in the previous case. Detectors based on the emission of charged capacitor with transparent for radiation plates in vacuum was considered in [36].

3.4 *Detectors based on conversion of gravitons into photons in a transverse magnetic field*

The conversion of the gravitational radiation into electromagnetic one can be produced in the conversion scheme which is reverse one to the scheme presented in the Fig.2 [4], [37]. The gravitational radiation in this scheme propagates in the direction of axis "y" and is converted to an electromagnetic radiation in the transverse magnetic field and in the electromagnetic field stored in the open resonator. The conversion coefficient in this case is

$$p_{g\to\gamma} = \frac{\dot{N}_r^{em}}{\dot{N}^{gr}} = \frac{G}{c^4}Q^2L^2B_\perp^2. \tag{16}$$

where $\dot{N}_r^{em}$ is the flow of photons stored in the resonator.

The quality of a resonator Q^2 in (16) takes into account stimulation of the conversion of the gravitational radiation by the resonator. At that the extraction of the energy from the gravitational wave will be increased Q times. The number of the photons stored in the resonator for the time $\Delta t > \tau$ is

$$\Delta N^{em} = \dot{N}_r^{em}T = \frac{2G}{c^5}Q^2L^2L_mB_\perp^2\dot{N}^{gr}, \tag{17}$$

where $\tau = QT$ is the rising time (increment) of this system, $T = 2L_m/c$ double time of the light passage between mirrors of the resonator.

In the case of $B_\perp = 10^5\,\mathrm{G}$, $L_m \simeq L = 1\,\mathrm{km}$, $P_a^{em} = 10^2\,\mathrm{W}$, $Q = 10^9$, $P_r^{em} = 10^{11}\,\mathrm{W}$, $P^{gr} = 8.26\cdot10^{-19}\,\mathrm{W}$, $\dot{N}^{gr} = 4\cdot10^4\,\mathrm{gr/s}$, $\hbar\omega_{gr} = 1.25\cdot10^{-4}\,\mathrm{eV}$ or $\lambda = 1\mathrm{cm}$ according to (16) the value $\dot{N}_r^{em}/\dot{N}^{gr} = 8.26\cdot10^{-12}$, the flow of converted photons in the resonator $\dot{N}_r^{em} \simeq 3.3\cdot10^{-7}$ ph/s, the number of the photons stored in the resonator for the time $\Delta t > \tau = 6.7\cdot10^3\,\mathrm{ms}$ is $\Delta N^{em} = 2.2\cdot10^{-8}$. This is very small value. Of cause we can take the length of the resonator, the value of the magnetic field and the flow of the gravitons much higher [31], [38]. In this case the flow of gravitons can achieve an accessible value (some photons per a day).

3.5 *Stimulation of the conversion of gravitons into photons by an open resonator*

When an oscillator emit in an open resonator an electromagnetic wavepacket then a part of this wavepacket will be stored in transverse and longitudinal modes. After reflections from the mirrors of the resonator the amplitude $E_{\alpha0}$ of the emitted wavepacket will be decreased $(r_lr_r)^n$ times, where r_i^2 are the reflectivity coefficients of the left l and right r mirrors accordingly, n the number of reflections from the pair of mirrors. When oscillator emit a train of wavepackets with a period T then after N reflections of the first wavepacket

the amplitude of the total electric field strength of wavepackets stored in the resonator will be determined by the expression

$$E_{\alpha}^{\Sigma} = \sum_{n=0}^{N} E_{\alpha 0}(r_l r_r)^n = E_{\alpha 0} Q(1 - (r_l r_r)^N) \tag{18}$$

where the value $Q = 1/(1 - r_l r_r)$ is the quality of the resonator. The value $N \simeq t/T$ is the number of reflections of the first wavepacket for the time interval t, the value $(r_l r_r)^N \simeq \exp[-t/2\tau]$, where $\tau = -T/\ln{(r_l r_r)^2}|_{Q \gg 1} \simeq T\,Q/2$.

The energy stored in the resonator will decay by the low: $\varepsilon^{em} = \varepsilon_0^{em} \exp(-t/\tau)$.

In the case of the main transverse mode in the resonator TEM_{00} the density of the electromagnetic energy in the mode will have Gaussian form. The power of the electromagnetic radiation in the mode will be determined by the product of the Poynting's vector for the intensity $I = (c/8\pi)E_m^2$ and the area of the mode $\pi\sigma_0^2$, where E_m is the amplitude of the linear polarized wave. In practical unites $I = (1/240\pi)E_m^2 \simeq 1.33 \cdot 10^{-3} E_m^2\,[\mathrm{W/V^2}]$. The maximum average intensities of the radiation reflected from the mirrors $I = 1.33{\cdot}10^7\ W/cm^2$ and $I = 1.33 \cdot 10^9$ W/cm ^{2} are limited by heating of mirrors and correspond to the electric field strengths accordingly $E_m \simeq 10^5$[V/cm] for the case of warm metal mirrors and to $E_m \simeq 10^6$[V/cm] for the superconducting mirrors in a cm wavelength region.

4 On a possibility of laboratory gravitational experiments

The combined symmetrical scheme of an experiment based on double conversion of electromagnetic to gravitational radiation and back is presented on Fig.4. In this scheme two equal semi-confocal resonators of the length L_m each are presented by mirrors $M1$ - $M3$. $M3$ is a thick flat mirror, which is simultaneously the absorber of the powerful electromagnetic radiation exited in the left resonator.

In this case the ratio of the reactive power of the electromagnetic radiation stored into the right resonator to the active power of electromagnetic radiation supplying the left resonator and the number of photons stored in the right resonator are

$$p_{\gamma\to g\to\gamma} = \frac{P_{r\,r}^{em}}{P_{a\,l}^{em}} = \frac{\dot{N}_{r\,r}^{em}}{\dot{N}_{a\,l}^{em}} = \frac{G^2}{c^8} Q_l Q_r^2 L^4 B_{\perp}^4, \tag{19}$$

$$\Delta N_r^{em} = p_{\gamma\to g\to\gamma} \dot{N}_{a\,l}^{em}\, T, \tag{20}$$

where $G^2/c^8 \simeq 6.78 \cdot 10^{-99}$, Q_l and Q_r are the qualities of the left and right resonators accordingly.

Example 3. Let us the qualities of the superconducting resonators are $Q_1 = Q_2 = 10^9$, the value of the transverse magnetic fields $B_{\perp} = 10^6$ Gs, the lengths of the magnet $L = 2{\cdot}10^6$ cm, the distance between mirrors $M1$ - $M3$ and $M3$ $M2$ $L_m \simeq L/2 = 10^6$ cm, $\hbar\omega_{gr} = 1.25 \cdot 10^{-4}$ eV ($\lambda = 1$cm), the intensity of the electromagnetic wave stored in the resonator $I = 1.33 \cdot 10^9[\mathrm{W/cm^2}]$, the active power supplying the left resonator $P_{a\,l}^{em} = 6.62 \cdot 10^5$W.

In this case the reactive power stored in the resonator $P_{a\,l}^{em} = 6.62 \cdot 10^{14}$W, the flow of photons from the laser supplying the left resonator $\dot{N}_{a\,l}^{em} = 3.31 \cdot 10^{28}$ ph/s, the flow

of photons stored in the left resonator $N^{em}_{a\,l} = 3.31 \cdot 10^{37}$ ph/s, and according to (19) the ratio $p_{\gamma\to g\to\gamma} = 6.78 \cdot 10^{-24}$, $\sigma_0 = 398\,$cm, $T = (2/3) \cdot 10^{-5}\,$s, $\tau = (2/3) \cdot 10^4\,s\simeq$ 2 hours, $\Delta N^{em} = 0.748\,gr$, the flow of photons $\dot{N}^{em}_{r,r} = 2.24 \cdot 10^{-4}$ ph/s or $\sim 1\,$ph/hour. At that in the right resonator the flow of gravitons is $\dot{N}^{gr}_r = 2.72 \cdot 10^{12}$ gr/s.

The number of photons stored in the right resonator $\Delta N^{em} \simeq 1$ or the flow of photons $\dot{N}^{em}_{r,r} \sim 1\,$ph/hour are not so small to reject the possibility of the gravitational experiments in principle. The value of the magnetic field $\sim 10^6$ G is not reached now on experiment but it does not exceed the theoretical limit. The quality of the open resonator $Q \sim 10^{11}$ was achieved in the open resonator in the decimeter wavelength region. The diameter of the superconducting mirror $\sim$10m, the volume of the superconducting magnet ($10^6\,$m^3), and the average power of a laser or a free-particle laser $\sim 1\,$MW are very huge and expensive but capable of execution.

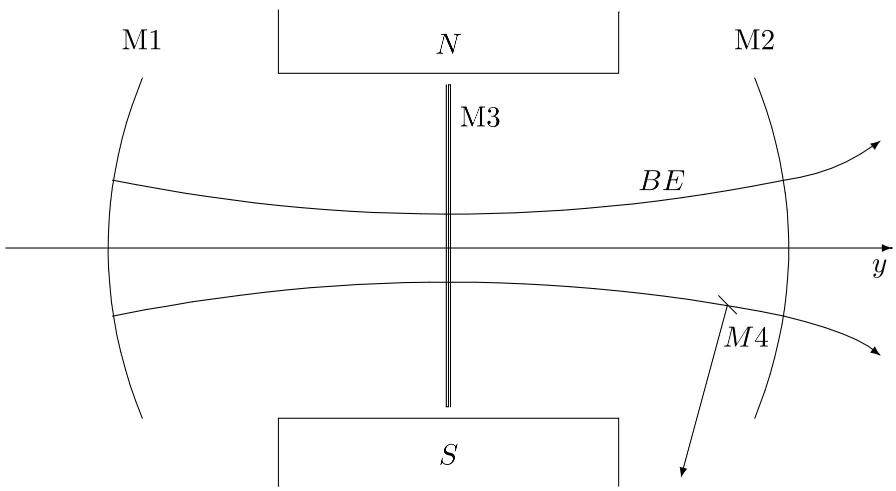

Fig.4: A scheme of conversion of an electromagnetic laser beam exited in an open resonator into gravitational one and back; $M1$, $M2$, $M3$, $M4$: Mirrors; N, S: magnetic poles; BE: Beam Envelops of an electromagnetic and gravitational radiations.

The efficiency of the direct ways of the graviton production and detection are less.

5 Conclusion

The direct way of emission of gravitational waves by charged ions nonuniformly moving in external electromagnetic or gravitational fields and conversion of the electromagnetic radiation into gravitational one and back in the transverse magnetic fields have a different nature. Both of them are in the Einstein's theory of gravity [39]. Nevertheless second one is not so obvious. That is why they are need personal verification in the emission and conversion processes. The lack of the conversion process will not mean the lack of the gravitational radiation emitted by the nonuniformly moving particles.

The power of any gravitational radiation source is very small and an interaction of gravitational waves with any feasible detector is very small. This is the reason why people

were not able to confirm the existence of such radiation. Nowadays the experimental efforts are concentrated on the detection of Gravitational Radiation coming from very intense astrophysical sources, such as Supernovae explosions, by means of cryogenic resonant bars and interferometers. Artificial sources of gravitational waves under human control will be the next step in an accurate study the nature of the gravitational radiation, its connection with another theories and its unification with other forces. An experiment of laboratory production and detection of gravitational waves would justify efforts comparable with those done for particle accelerators. This happened in particle physics after the introduction of accelerators beside of the use of cosmic ray detectors.

Unfortunately the intensity we can produce with the artificial gravitational radiation sources proposed until now is much smaller then requested by the sensitivity of the possible detectors actually conceived. This could discourage on the possibility to realize an experiment of production and detection of gravitational waves in laboratory within not extremely long times. However some people are optimist, and think that such an experiment will be done within the next century. For this reason it is important to continue in conceiving, studying and improving ideas for possible man made sources of gravitational waves and for suitable detectors. If one rely on the rapid technological increasing and on the continuous flow of new ideas, it can hope the man will be able to do an experiments of such kinds within next century [31].

We hope that using of Grasers based on cold heavy ion beams (or more heavy charged formations) and cutoff waveguides or on conversion of electromagnetic waves in the transverse magnetic field as well as using detectors with stimulation of graviton conversion by high quality resonator and maybe another ideas (quantum nondemolition measurements [40], using amplification by active media and so on [41], [42]) instill an additional hope in the reality of the experiments on the verification of the gravitational theories.

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